AL-FARABI KAZAKH NATIONAL UNIVERSITY

**Faculty of Mathematics and Mechanics**

# Differential Equations and Control Theory Department

**TASKS AND METHODICAL HELP FOR THE PRACTICAL WORKS**

**Architecture of Mathematics**

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**Module 2. Sets**

**Examples of relations and operators**

Week 3

**Examination 1**

Give examples of notions if it exists:

1. Reflexive symmetric, but no transitive relation.

2. Surjection from the set of squares to the set of segments.

**Examination 2**

Give examples of notions if it exists:

1. Operator, which is not surjection and injection ей.

2. Reflexive no symmetric relation of ellipses.

**Examination 3**

Give examples of notions if it exists:

1. Transitive relation of triangles.

2. Surjection from the set of squares to the set of positive numbers.

**Examination 4**

Give examples of notions if it exists:

1. Symmetric no transitive relation.

2. Equivalence of two subsets of set of complex numbers.

**Examination 5**

Give examples of notions if it exists:

1. Bijection between sets of people and real numbers.

2. Reflexive transitive, but no symmetric relation.

**Examination 6**

Give examples of notions if it exists:

1. Injection from the set of triangles to the set of parallelograms.

2. Correspondence between sets of real numbers and lines.

**Examination 7**

Give examples of notions if it exists:

1. Transitive no symmetric relation of complex numbers.

2. Bijection between subset of sets of real numbers and dogs.

**Examination 8**

Give examples of notions if it exists:

1. Reflexive transitive relation of squares.

2. Injection from the set of real numbers to the set of segments.

**Examination 9**

Give examples of notions if it exists:

1. Surjection between the sets of triangles and negative numbers.

2. Reflexive transitive relation of segments.

**Examination 10**

Give examples of notions if it exists:

1. Reflexive symmetric, but no transitive relation between people.

2. Injection from the set of sides of a trapezium to the set of its vertexes.

**Examination 11**

Give examples of notions if it exists:

1. Relation between people, which is not reflexive.

2. Bijection between sets of real and rational numbers.

**Examination 12**

Give examples of notions if it exists:

1. Reflexive symmetric relation of lines.

2. Surjection from the set of numbers to the set of balls.

**Examination 13**

Give examples of notions if it exists:

1. Reflexive no symmetric relation of parabolas.

2. Bijection between sets of sides of a square and cube.

**Examination 14**

Give examples of notions if it exists:

1. Injection from the set of vertexes of cube to the set of natural numbers.

2. Transitive no symmetric relation of squares.

**Examination 15**

Give examples of notions if it exists:

1. Injection from the set of натуральных numbers to the set of вершин куба.

2. Correspondence between sets of sides and vertexes of a rectangle.

**Examination 16**

Give examples of notions if it exists:

1. Symmetric no reflexive relation of curves.

2. Injection from the set of natural numbers to the set of real numbers.

**Examination 17**

Give examples of notions if it exists:

1. Transitive no reflexive relation людей.

2. Surjection from the set of natural numbers to the set of real numbers.

**Examination 18**

Give examples of notions if it exists:

1. Reflexive symmetric relation of triangles.

2. Bijection between sets of rational and real numbers.

**Module 3. Numbers**

Week 5

**Examination 1**

1. Give an example of the object if it exists: injection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 2**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity of the addition on the set Z, if this property is true on the set N.

**Examination 3**

1. Give an example of the object if it exists: surjection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 4**

1. Give an example of the object if it exists: injection .

2. Prove the associativity of the multiplication on the set Q, if this property is true on the set Z.

**Examination 5**

1. Give an example of the object if it exists: surjection .

2. Prove the commutativity of the addition on the set R, if this property is true on the set Q.

**Examination 6**

1. Give an example of the object if it exists: surjection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 7**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 8**

1. Give an example of the object if it exists: injection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 9**

1. Give an example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set C, if this property is true on the set R.

**Examination 10**

1. Give an example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 11**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 12**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 13**

1. Give an example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 14**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 15**

1. Give an example of the object if it exists: surjection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 16**

1. Give an example of the object if it exists: surjection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 17**

1. Give an example of the object if it exists: injection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 18**

1. Give an example of the object if it exists: injection .

2. Prove the associativity of the addition on the set Z, if this property is true on the set N.

**Module 4. Ordered objects**

**Examples and properties of ordered sets**

Week 7.

**Examination 1**

Give an example of objects:

1. The set with largest element without smallest element.

2. The quasi ordered set of people.

**Examination 2**

Give an example of objects:

1. The linear ordered non numerical set.

2. The set, which is lower bounded but not upper bounded.

**Examination 3**

Give an example of objects:

1. The set with two maximal elements.

2. An ordered set of circles.

**Examination 4**

Give an example of objects:

1. The set with two minimal elements.

2. The quasi ordered set of triangles.

**Examination 5**

Give an example of objects:

1. The linear ordered set, which is not completely ordered.
2. The set with three minimal elements.

**Examination 6**

Give an example of objects:

1. The bounded set of circles.

2. The monotone operator on non numerical sets.

**Examination 7**

Give an example of objects:

1. The set, which is upper bounded but not lower bounded.

2. Anti monotone operator on non numerical sets.

**Examination 8**

Give an example of objects:

1. Unbounded set of circles.

2. Operator, which is monotone and anti monotone.

**Examination 9**

Give an example of objects:

1. The set with a smallest element without a largest element.
2. Operator, which is not monotone and anti monotone.

**Examination 10**

Give an example of objects:

1. Interval on the ordered set of balls.

2. The quasi ordered set of elephants.

**Examination 11**

Give an example of objects:

1. The isomorphism of orders on non numerical sets.

2. Unbounded set of triangles.

**Examination 12**

Give an example of objects:

1. The set with two largest elements.

2. The quasi ordered set of vectors.

**Examination 13**

Give an example of objects:

1. The set with two maximal elements and one minimal element.

2. The quasi ordered set of ellipses.

**Examination 14**

Give an example of objects:

1. The set without largest and smallest elements.

2. The quasi ordered set of balls.

**Examination 15**

Give an example of objects:

1. The set with two minimal elements and one maximal element.

2. The isomorphism of order on non numerical sets.

**Examination 16**

Give an example of objects:

1. Bounded set of circles.

2. The set with three maximal elements.

**Module 5. Algebraic objects**

**Examples and properties of algebraic objects**

Week 9

**Examination 1**

1. Determine a semigroup on the set continuous functions.

2. Determine a linear functional on the set of integrable functions.

**Examination 2**

1. Determine a monoid on the set of two elements.

2. Determine a linear space on the set of trigonometric polynomials.

**Examination 3**

1. Determine a groupoid but not monoid on the set of three elements.

2. Determine a linear space, which is isomorphs to .

**Examination 4**

1. Determine a but not group on the set negative numbers.

2. Determine a linear space on the set of polynomials.

**Examination 5**

1. Determine a commutative monoid on the set of polynomials.

2. Determine a linear operator from the plane to the set of continuous functions.

**Examination 6**

1. Determine a group on the set of circles.

2. Determine a linear operator from the set of numbers to the set of continuous functions.

**Examination 7**

1. Determine non commutative groupoid on the set of two elements.

2. Determine a linear operator from the set of continuous functions to the plane.

**Examination 8**

1. Determine a semigroup on the set of even numbers.

2. Determine a linear functional on the set differentiable functions.

**Examination 9**

 1. Determine a monoid but not a group on the set of three elements.

2. Determine a subspace of the space of complex numbers.

**Examination 10**

1. Determine a groupoid on the set camels.

2. Determine a linear operator from the plan to the space.

**Examination 11**

1. Determine a non commutative groupoid on the set of sides of triangles.

2. Determine a non convex set of complex numbers.

**Examination 12**

1. Determine a group on the set множестве piecewise constant functions.

2. Determine a convex set of complex numbers.

**Examination 13**

1. Determine a non commutative groupoid on the set of tops of triangles.

2. Determine a one-dimensional subspace of continuous functions.

**Examination 14**

1. Determine a non commutative groupoid on the set of two elements and its dual groupoids.

2. Determine a linear operator from the space to the plane.

**Examination 15**

1. Determine a non commutative groupoid on the set of continuous functions.

2. Determine a finite dimensional subspace of the space of continuous functions.

**Examination 16**

1. Determine commutative groupoid on the set of two elements.

2. Determine a convex set of continuous functions.

**Module 6. Topological objects**

**Examples and properties of topological objects**

Week 11

**Examination 1**

1. Determine a weakest topology on the set of three points. Find neighbourhoods of all points.

2. Determine a disconnected set with connected boundary on the plane.

**Examination 2**

1. Determine a strongest topology on the set of three points. Find neighbourhoods of all points.

2. Determine a set with open boundary on the line.

**Examination 3**

1. Determine a connected topology on the set of three points.

2. Determine two sets on the plane, which are equivalent with respect to the set theory but not to topology.

**Examination 4**

1. Determine a non connected topology on the set of three points.

2. Is it possible a set of the line be equivalent to an open set with respect to the set theory? Explain it if it is impossible. Give an example if it is possible.

**Examination 5**

1. Determine two equivalent sets on the line, one of them is closed and second is not closed.

2. Determine a metric on the set of three elements.

**Examination 6**

1. Determine an inseparable topology on the set of three points.

2. Determine a set on the line with three points of boundary.

**Examination 7**

1. Determine a topology on the set of three points. Find all open, closed and other sets.

2. Determine a two connected but not homeomorphic sets.

**Examination 8**

1. Determine a topology on the set of three points, which is not discrete and indiscrete.

2. Determine a metric on the set bounded functions.

**Examination 9**

 1. Determine a non connected set on the plane with non bounded boundary if it is possible. Explain the situation if it is impossible.

2. Is it possible to determine a metric on the set of three elements with unbounded set? Explain it if it is impossible. Give an example if it is possible.

**Examination 10**

1. Determine two topologies on the set of three points. Determine a sequence, which converges in first topology and diverges in the second one.

2. Determine a non connected closed set on the line.

**Examination 11**

1. Determine two closed sets of the line, which are non equivalent.

2. Determine a non connected bounded set of the line. Find its diameter.

**Examination 12**

1. Determine a separable topology on the set of three elements. Determine its converged sequence.

2. Determine a metric on the set of complex numbers. Determine the bounded set there. Find its diameter.

**Examination 13**

1. Determine two equivalent sets of the line, one of them is open and second is not open.

2. Determine a metric on the set of four elements.

**Examination 14**

1. Is the equivalence of sets a topological property? Explain this situation.

2. Determine a metric on the set of elephants.

**Examination 15**

1. Determine a non connected set on the plane with boundary, which contain one point.

2. Determine a metric on the set of circles.

**Examination 16**

1. Determine a separable topology on the set of three points.

2. Determine a non connected open set on the line.

**Module 6. Measurable** **objects**

**Examples and properties of measurable** **topological objects**

Week 13

**Examination 1**

1. Determine a measurable space on the circle, which is algebra.

2. Determine a measure on the set of four points.

**Examination 2**

1. Determine a σ-algebra on the set of four points.

2. Determine Lebesgue measure of a non connected closed set of the plane.

**Examination 3**

1. Determine a ring on the set of four points, which is not σ-algebra.

2. Determine a measure a on the set of circles.

**Examination 4**

1. Determine a measurable space on the set of squares.

2. Determine Lebesgue measure of a semi open interval.

**Examination 5**

1. Determine algebra on the set of sides of squares.

2. Determine a probabilistic measure on the set four points.

**Examination 6**

1. Determine a measurable space on the set of three points, which is not a topological space.

2. Determine a measure on a non connected open set of the plane.

**Examination 7**

1. Determine a measurable space on the set of elephants.

2. Find of Lebesgue measure of a non closed connected set of the line.

**Examination 8**

1. Determine a σ-algebra on the set of five points.

2. Is it possible a discontinuous operator, which saves a measure? Explain it if it is impossible. Give an example if it is possible.

**Examination 9**

 1. Is it possible algebra on the sides of a square, which is not σ-algebra? Explain it if it is impossible. Give an example if it is possible.

2. Determine a measure on the set of line with boundary, which has three points.

**Examination 10**

1. Determine a σ-algebra on the boundary of a square.

2. Determine Lebesgue measure of on the boundary of a semi open interval.

**Examination 11**

1. Determine σ-algebra on the set of tops of triangle.

2. Determine Lebesgue measure Лебега of an open unbounded interval on the line.

**Examination 12**

1. Determine algebra but not σ-algebra on the subset of the line with boundary, which contains one point, if it is possible. Explain it if it is impossible.

2. Determine a discontinuous operator on the plane, which save Lebesgue measure.

**Examination 13**

1. Is it possible to determine a ring but not algebra on the set with three points? Explain it if it is impossible. Give an example if it is possible.

2. Determine a measure on the set with four points.

**Examination 14**

1. Is it possible to determine algebra but not σ-algebra on the set with three points? Explain it if it is impossible. Give an example if it is possible.

2. Determine a continuous operator on the line, which does not save Lebesgue measure.

**Examination 15**

1. Determine algebra on disconnected set on the line with boundary, which has a two points.

2. Determine Lebesgue measure on the closed set of the line.

**Examination 16**

1. Determine a measurable space on the set of two points, which is not a topological space.

2. Determine a measure on the set of line with boundary, which has one point.

**Task 2. Operators**

**Give examples of notions if it exists.**

1. Surjection from the set of squares to the set of segments. Operator from the set of natural number to the set of real numbers that is not bijection.
2. Injection from the set of people to the set of trees. Operator from the set of positive number to the set of complex numbers that is not surjection.
3. Surjection from the set of squares to the set of positive numbers. Operator from the set of positive number to the set of complex numbers that is not injection.
4. Bijection between two subsets of set of complex numbers. Operator from the set of people to the set of cars that is not injection.
5. Bijection between sets of people and all real numbers. Operator from the set of birds to the set of complex numbers that is not surjection.
6. Injection from the set of triangles to the set of parallelograms. Operator between two subsets of set of complex numbers that is not surjection and injection.
7. Bijection between subset of sets of all real numbers and dogs. Operator from the set of students to the set of functions that is not surjection.
8. Injection from the set of real numbers to the set of segments. Operator from the set of all natural numbers to the set of boats that is not surjection.
9. Surjection between the sets of triangles and negative numbers. Operator from the set of people to the set of cars that is not injection.
10. Injection from the set of all sides of a trapezium to the set of all its vertexes. Operator from the set of dogs to the set of all integer numbers that is not surjection.
11. Bijection between sets of real and rational numbers. Operator from the set of all negative numbers to the set of complex numbers that is not injection.
12. Surjection from the set of numbers to the set of balls. Operator from the set of stars to the set of birds that is not surjection.
13. Bijection between sets of sides of a square and a cube. Operator from the set of all complex numbers to the set of people that is not injection.
14. Injection from the set of vertexes of cube to the set of natural numbers. Operator from the set of intervals to the set of real numbers that is not bijection.
15. Injection from the set of all natural numbers to the set of rivers. Operator from the set of all real number to a set of complex numbers that is not bijection.
16. Injection from the set of natural numbers to the set of real numbers. Operator from a set of tables to the set of all real numbers that is not surjection.
17. Surjection from the set of natural numbers to the set of real numbers. Operator from the set of lines to the set of curves that is not injection.
18. Bijection between sets of rational and real numbers. Operator from a set of functions to the set of complex numbers that is not injection.

**Task 3. Numbers (cardinalities and solutions)**

1. Give an example of the object if it exists: injection  that is not surjection. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.
2. Give an example of the object if it exists: injection  that is not surjection. Prove the associativity of the addition on the set Z, if this property is true on the set N.
3. Give an example of the object if it exists: surjection that is not injection. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.
4. Give an example of the object if it exists: injection  that is not surjection. Prove the associativity of the multiplication on the set Q, if this property is true on the set Z.
5. Give an example of the object if it exists: surjection  that is not injection. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.
6. Give an example of the object if it exists: bijection between N and Q. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.
7. Give an example of the object if it exists: surjection  that is not injection. Prove the associativity addition on the set Z, if this property is true on the set N.
8. Give an example of the object if it exists: injection  that is not surjection. Prove the associativity addition on the set Q, if this property is true on the set Z.
9. Give an example of the object if it exists: injection that is not surjection. Prove the commutativity of the addition on the set Z, if this property is true on the set N.
10. Give an example of the object if it exists: injection  that is not surjection. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.
11. Give an example of the object if it exists: surjection  that is not injection. Prove the associativity addition on the set Z, if this property is true on the set N.
12. Give an example of the object if it exists: surjection  that is not injection. Prove the distributivity on the set Z, if this property is true on the set N.
13. Give an example of the object if it exists: injection  that is not surjection. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.
14. Give an example of the object if it exists: surjection  that is not injection. Prove the associativity addition on the set Z, if this property is true on the set N.
15. Give an example of the object if it exists: surjection  that is not injection. Prove the distributivity on the set Q, if this property is true on the set Z.
16. Give an example of the object if it exists: injection  that is not surjection. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.
17. Give an example of the object if it exists: injection  that is not surjection. Prove the associativity addition on the set Z, if this property is true on the set N.
18. Give an example of the object if it exists: injection  that is not surjection. Prove the associativity of the addition on the set Z, if this property is true on the set N.

**Task 4. Quasiordered sets Numbers**

Determine examples of the quasiorder on the following sets.

1. Set of curves, set of cars.
2. Set of triangles, set of trees.
3. Set of balls, set of people.
4. Set of functions, set of cats.
5. Set of squares, set of dogs.
6. Set of polynomials, set of flowers.
7. Set of equations, set of bridges.
8. Set of circles, set computers.
9. Set of rectangles, set ships.
10. Set of ellipses, set of airplanes.
11. Set of cones, set of birds.
12. Set of diameters of a circle, set of houses.
13. Set of cubes, set of rivers.
14. Set of trigonimetric functions, set of seas.
15. Set of sectors of a circle, set of words.
16. Set of trapeziums, set of cities.
17. Set of lines on the plane, set of apples.
18. Set of planes, set of pencils.

**Task 5. Ordered objects (examples and properties)**

Give an example of the ordered objects

1. The set with largest element without smallest element; the bounded set of people.
2. The linear ordered non-numerical set; the set, which is lower bounded, but not upper bounded.
3. The set with two maximal elements; the ordered set of circles.
4. The set with two minimal elements; the unbounded set of triangles.
5. The linear ordered set, which is not completely ordered; the set with three minimal elements.
6. The bounded set of circles; the monotone operator between non-numerical sets.
7. The set that is upper bounded, but not lower bounded; antimonotone operator between non- numerical sets.
8. The unbounded set of circles; the operator that is monotone and antimonotone.
9. The set with a smallest element without a largest element; operator that is not monotone and antimonotone.
10. The interval on the ordered set of balls; the set with two minimal and two maximal elements.
11. The unbounded set of curves; the non-numerical set without largest and smallest elements.
12. The set of vectors with largest element; the interval on the non-numerical set.
13. The set with two maximal elements and one minimal element; two orders on the same non-numerical set.
14. The set without largest and smallest elements; the unbounded set of balls.
15. The set with two minimal elements and one maximal element; the monotone operator between non-numerical sets.
16. The bounded set of circles; the set with three maximal elements.
17. The upper unbounded set of people; the set with three minimal elements.
18. The lower unbounded set of people; the set with two minimal elements.